General Certificate of Education (A-level) January 2011

## Mathematics

MPC1

## (Specification 6360)

Pure Core 1

Mark Scheme

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## Key to mark scheme abbreviations

| M | mark is for method |
| :--- | :--- |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| Jor ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0 ) accuracy marks |
| $-x$ EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied <br> SCA |
| cubstantially correct approach |  |
| cf | candidate |
| dp | significant figure(s) |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award full marks. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn no marks.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns full marks, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains no marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 1(a) | $\frac{\mathrm{d} y}{\mathrm{~d} x}=18+6 x-12 x^{2}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \end{aligned}$ | 3 | one of these terms correct another term correct all correct (no $+c$ etc) (penalise $+c$ once only in question) |
| (b) | $18+6 x-12 x^{2}=0$ | M1 |  | putting their $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$, PI by attempt to solve or factorise |
|  | $6(3-2 x)(x+1) \quad(=0)$ | m1 |  | attempt at factors of their quadratic or use of quadratic equation formula |
|  | $x=-1, x=\frac{3}{2} \quad \text { OE }$ | A1 | 3 | must see both values unless $x=-1$ is verified separately <br> If M1 not scored, award SC B1 for verifying that $x=-1$ leads to $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ and a further SC B2 for finding $x=\frac{3}{2}$ as other value |
| (c)(i) | $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=6-24 x$ | B1〕 |  | FT their $\frac{\mathrm{d} y}{\mathrm{~d} x}$ but $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ must be correct if 3 marks earned in part (a) |
|  | When $x=-1, \frac{\mathrm{~d}^{2} y}{\mathrm{~d} x^{2}}=6-(24 \times-1)$ | M1 |  | Sub $x=-1$ into 'their' $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ |
|  | $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=30$ | A1cso | 3 |  |
| (ii) | Minimum point | E1 $\checkmark$ | 1 | must have a value in (c)(i) <br> FT "maximum" if their value of $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}<0$ |
|  | Total |  | 10 |  |

## MPC1 (cont)




| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 5(a)(i) |  | M1 A1 A1 | 3 | cubic curve with one max and one min (either way up) curve touching positive $x$-axis (either way up) <br> correct graph passing through $O$ and touching $x$-axis at 2 |
| (ii) | $\begin{aligned} & x\left(x^{2}-4 x+4\right)=3 \\ & \Rightarrow x^{3}-4 x^{2}+4 x-3=0 \end{aligned}$ | B1 | 1 | AG (must have $=0$ ) |
| (b)(i) | $\begin{aligned} \mathrm{p}(-1) & =(-1)^{3}-4(-1)^{2}+4(-1)-3 \\ & (=-1-4-4-3) \end{aligned}$ | M1 |  | $\mathrm{p}(-1)$ attempted (condone one slip) or full long division to remainder |
|  | $=-12$ | A1 | 2 | must indicate remainder $=-12$ if long division used |
| (ii) | $p(3)=3^{3}-4 \times 3^{2}+4 \times 3-3$ | M1 |  | $p(3)$ attempted (condone one slip) NOT long division |
|  | $p(3)=0 \Rightarrow x-3$ is factor | A1 | 2 | shown $=0$ plus statement |
| (iii) | Either $b=-1$ (coefficient of $x$ correct) or $c=1$ (constant term correct) | M1 |  | allow M1 for full attempt at long division or comparing coefficients if neither $b$ nor $c$ is correct |
|  | $\mathrm{p}(x)=(x-3)\left(x^{2}-x+1\right)$ | A1 | 2 |  |
| (c) | Discriminant of 'their quadratic' $=(-1)^{2}-4$ | M1 |  | numerical expression must be seen |
|  | Discriminant $=-3($ or $<0) \Rightarrow$ no real roots | A1cso |  | must have correct quadratic and statement and all working correct |
|  | (Only real root is $x=$ ) 3 | B1 | 3 |  |
|  | Total |  | 13 |  |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 6(a)(i) | $(x+3)^{2}+(y-1)^{2}$ | B1 |  | condone $(x--3)^{2}$ |
|  | $=13$ | B1 | 2 | $\text { condone }(\sqrt{13})^{2}$ |
| (ii) | $x^{2}+6 x+9+y^{2}-2 y+1$ | M1 |  | attempt to multiply out both of 'their' brackets; must have $x$ and $y$ terms |
|  | $\begin{array}{r} x^{2}+y^{2}+6 x-2 y \\ -3=0 \end{array}$ | $\begin{aligned} & \text { A1 } \\ & \text { A1 } \end{aligned}$ | 3 | both $m=6$ and $n=-2$ <br> All correct, $p=-3$ and..$=0$ |
| (b) | $x=0 \Rightarrow y^{2}-2 y-3=0$ | M1 |  | putting $x=0 \quad$ PI and attempt to solve or factorise |
|  | $\begin{aligned} & \Rightarrow(y-3)(y+1)=0 \\ & y=3, y=-1 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |  | and attempt to solve or factorise |
|  | $\Rightarrow$ Distance $A B=3+1=4$ | A1cso | 3 | $\begin{array}{rlr} \text { OR Pythagoras } & d^{2}=13-3^{2} & \text { M1 } \\ d=2 & \text { A1 } \\ \text { distance } & =2 \times 2=4 & \text { A1 } \end{array}$ |
| (c)(i) | $(-5+3)^{2}+(-2-1)^{2}=4+9$ $=13$ |  |  | Substitution $x=-5, y=-2$ into any correct circle equation |
|  | $\Rightarrow D$ lies on circle | B1 | 1 | convincing verification plus statement |
| (ii) | $\operatorname{grad} C D=\frac{1+2}{-3+5}$ | M1 |  | condone one sign slip |
|  | $=\frac{3}{2} \quad(\text { or } 1.5)$ | A1 | 2 | $\text { not } \frac{-3}{-2}$ |
| (iii) | $\text { Perpendicular gradient }=-\frac{2}{3}$ | M1 |  | ft their grad $C D$ or $m_{1} m_{2}=-1$ stated |
|  | Tangent has equation $y+2=-\frac{2}{3}(x+5)$ | A1 | 2 | any form of correct equation <br> eg $2 x+3 y+16=0$ $y=-\frac{2}{3} x+c, c=-\frac{16}{3}$ |
|  | Total |  | 13 |  |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 7(a)(i) | (-) $(x+5)^{2}$ | M1 |  | $q=5$; condone $(-x-5)^{2}$ |
|  | $29-(x+5)^{2}$ | A1 | 2 | $p=29$ and $q=5$ |
| (ii) | $x=-5$ is line of symmetry | B1 $\sqrt{ }$ | 1 | FT $x=-$ 'their $q$ ' or correct |
| (b)(i) | $4-10 x-x^{2}=k(4 x-13)$ |  |  |  |
|  | $\Rightarrow x^{2}+4 k x+10 x-13 k-4=0$ |  |  | Must see both these lines OE |
|  | $\Rightarrow x^{2}+2(2 k+5) x-(13 k+4)=0$ | B1 | 1 | AG all correct working and = 0 |
| (ii) | 2 distinct roots $\Rightarrow b^{2}-4 a c>0$ | B1 |  | stated or used (must be $>0$ ) |
|  | $\begin{aligned} & \text { Discriminant }=4(2 k+5)^{2}+4(13 k+4) \\ & 4\left(4 k^{2}+20 k+25+13 k+4\right)>0 \end{aligned}$ | M1 |  | condone one slip (may be within formula) or $16 k^{2}+132 k+116>0$ |
|  | $\Rightarrow 4 k^{2}+33 k+29>0$ | A1 | 3 | AG > 0 must appear before final line |
| (iii) | $(4 k+29)(k+1)$ | M1 |  | correct factors or correct unsimplified quadratic equation formula $\frac{-33 \pm \sqrt{33^{2}-4 \times 4 \times 29}}{8}$ |
|  | $k=-\frac{29}{4}, k=-1$ | A1 |  | condone $k=-\frac{58}{8},-7.25$ etc but not left with square roots etc as above |
| $-\frac{29}{4}$ |  | M1 |  | sketch or sign diagram including values |
|  | $k<-\frac{29}{4}, \quad k>-1$ <br> Take their final line as their answer | A1 | 4 | condone use of OR but not AND |
|  | Total |  | 11 |  |
|  | TOTAL |  | 75 |  |

